

Fast Fourier Transform

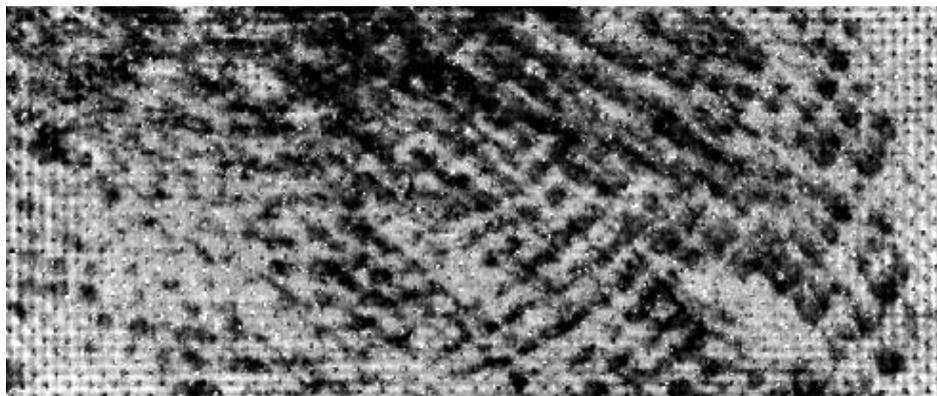
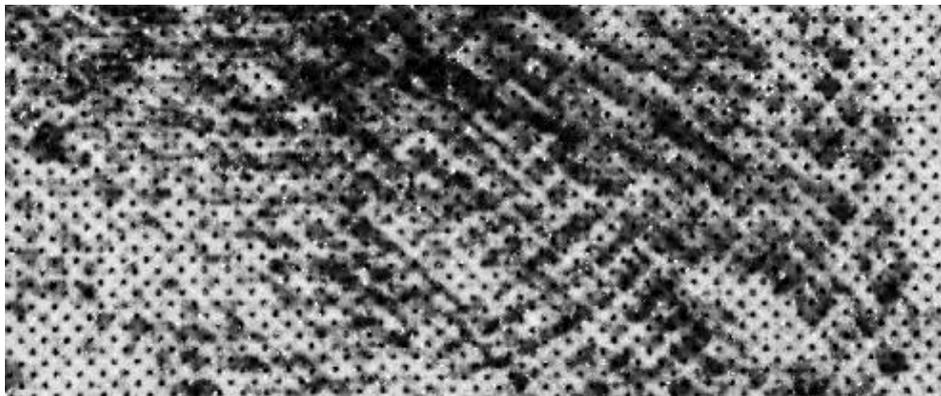
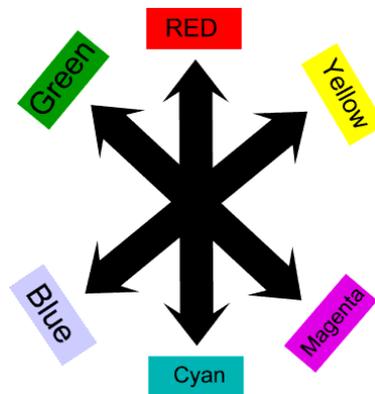


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The Fourier Transform

Fingerprints are developed on a wide diversity of surfaces, many of these contain background patterns, which interfere with the fingerprint characteristics and invalidate the fingerprint for evidential use. Examples of this would be fingerprints recovered on bank notes, printed documents, newspapers, magazines, map books, screen printed plastic and carrier bags etc. The Fourier transform enables us to isolate the wavelengths causing this problem and reduce their effect.



To help us to understand how the Fourier transform works, let's start by looking at the use of colour filters in black and white photography.

Look at the picture of the colour wheel at the top right of the screen here we have the primary colours of Red, Green and Blue, and the secondary colours of Yellow, Cyan and Magenta. When used in black and white photography each colour will transmit its self and absorb its opposites. Let's take the example of a red filter, this will absorb blue and green light and transmit red light. The film will subsequently become more sensitive to the red light and significantly less sensitive to the blue and green light. This means that the red parts of an object will appear lighter on the final print and the other colours will appear darker.

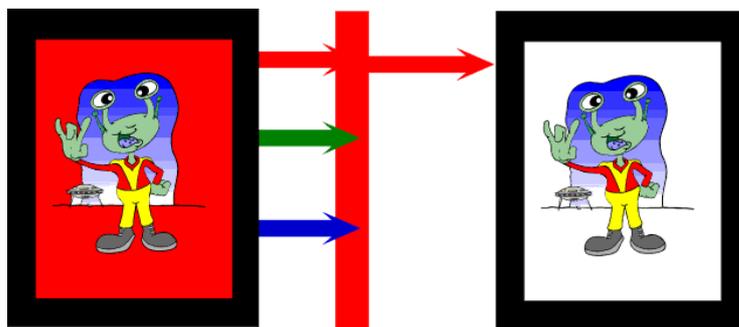
In fingerprint photography imagine what would happen if you had a black magenta powdered fingerprint impression on a red background. The final photographic print will portray a black fingerprint on a black background, to produce a print where the fingerprint is visible the photographer will need to use a red filter. The red filter will lighten the red to give a final print with a black fingerprint on a white background.

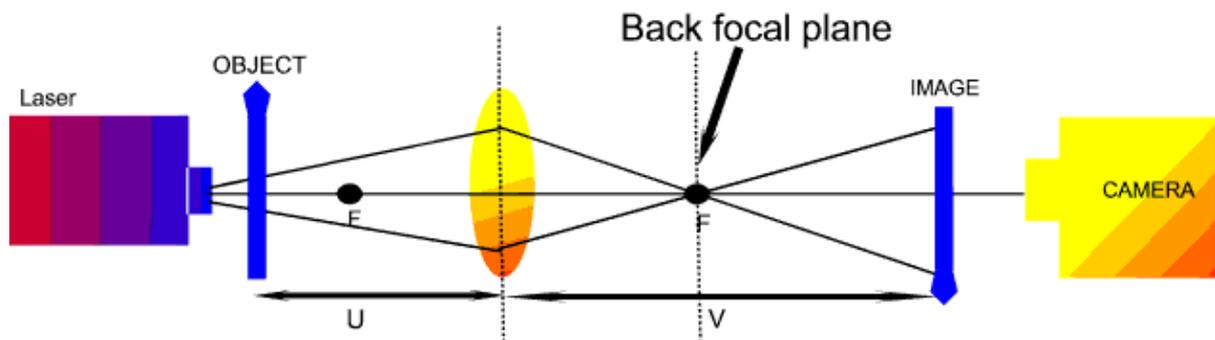
Alternatively if the photographer is photographing a Ninhydrin (magenta) fingerprint impression a green filter would be recommended, the filter will transmit green and absorb red and blue, this will have the effect of amplifying the magenta to produce a darker fingerprint impression. Effectively once the image has moved from a negative to a positive, it will have the visual effect of lightening its self and darkening its opposite.

Also this same technique is used to photograph plants and skies, if a green filter is used to photograph a plant the foliage will have a light colour and the flower will appear much darker, if a red filter is used to photograph a sky, the final print will show the sky almost black. This technique is often used to create dramatic if tonally distorted pictures, and to think someone once said the "camera never lies".

So in conclusion we can say that we have used colour filters to enhance our image and consequently improve the way it appears to the human eye. A black fingerprint on a black background is not very clear giving a low evidential value alternatively a black fingerprint on a white background has a high evidential value.

The Fourier transform is simply a means of being able to isolate a specific wavelength of light (which could not be removed with the use of colour filters) and reduce its effect in the final image. For example although we can use a red filter to lighten a background or level out the background on coca cola cans, a colour filter can not remove black or very dark lines or patterns. The Fourier transform on the other hand can. On the next page we will be looking at how we can perform a Fourier transform using a convex lens as the Fourier Transform computer.





The Convex lens as a Fourier Transform Computer

Fourier transform technology is based on scientific principles which have been available since the seventeenth hundreds. The original equations were developed by a French mathematician called [Jean Fourier \(1768-1830\)](#). His work is commonly known as the Fourier approach to diffraction, and is the basis of the principle known as Fourier optics. The Fourier principle adheres to the following concepts :-

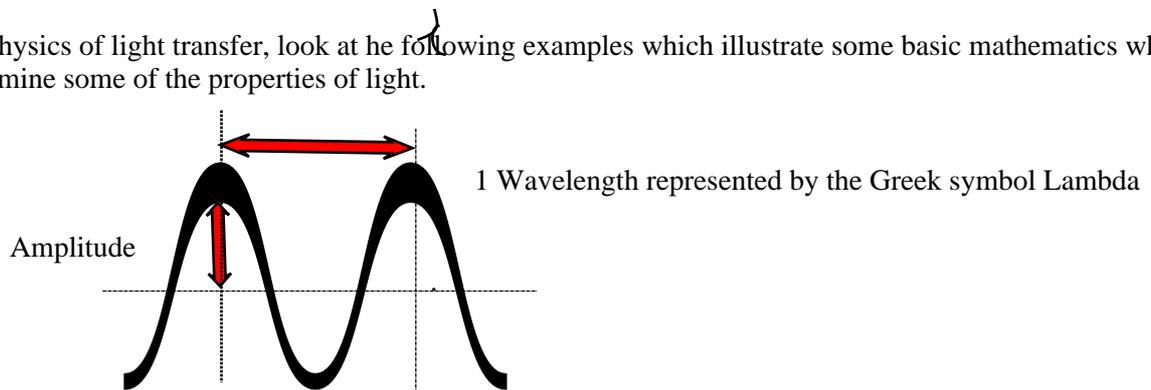


[Jean Fourier 1768-1830](#)

Any infinitely periodic profile can be analysed into (or synthesised from) a harmonic series of sinusoidal curves of appropriate amplitude.

It based on the physics of light transfer, look at the following examples which illustrate some basic mathematics which are used to determine some of the properties of light.

The Wavelength



Wavelengths of light have many properties the fundamentals of which are the waves PERIOD, Frequency and SPEED. The period of a wave is calculated via the equation $T = \text{Lambda}/v$ ($v =$ speed of the wave.) The frequency of the wave is calculated from the equation $f = 1/T$ ($f =$ the frequency of the wave and T the period of the wave.) Finally the speed of the wave is calculated from the equation $v = f/\text{Lambda}$, that is to say the speed of the wave is equal to the product of it's frequency and it's wavelength.

Transmittance amplitude

If we think of light as electromagnetic energy and represent it by the letter E we can have E_{in} and E_{out} (electromagnetic energy input, and electromagnetic energy output). All objects have what is called their transmittance amplitude, this is the amount of light they transmit, that is to say when light hits an object and passes through the amount of light which the object transmits is known as the transmittance amplitude of the object. This output or indeed the input or transmittance amplitude it's self can all be calculated mathematically with the following equations.

Transmittance amplitude is calculated by the following equation $E_{in}/E_{out} = T_a$ (transmittance amplitude.)

Lens Equation

In the above diagram the convex lens acts as the fourier transform computer. Light travels in straight lines and is only bent when it comes into contact with or passes through a substance of a lesser or greater density, this bending would be referred to as the refractive index of the substance. When light rays are passed through a convex lens they bend away from the apex to form a focal point, these distances between the image and the object relate directly to the focal length of the lens and are related to each other in the following lens equation

$$1/v + 1/U = 1/f$$

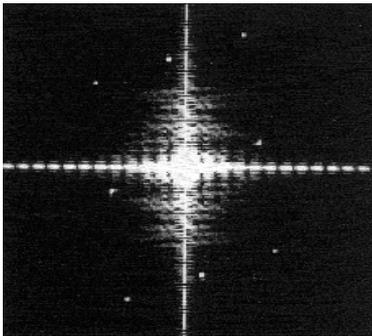
V represents the lens to image distance, U is the object to lens distance and F is the focal length of the convex lens.

The Diffraction pattern

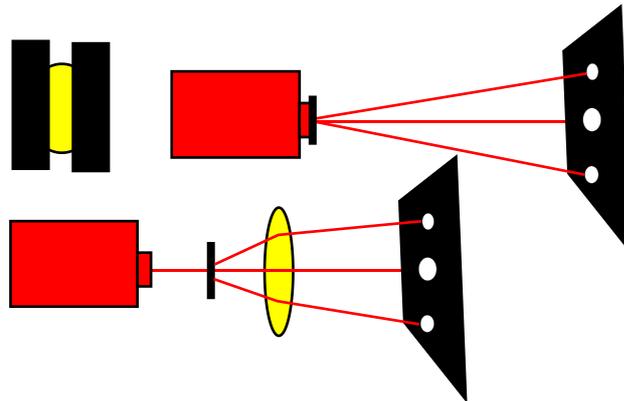
Lenses have both a front focal point and a back focal point (plane) our eyes also have a front focal point (usually 250mm) move your hand towards your eyes and at some point your hand will go out of focus, it therefore follows that all object must be placed in front of the focal point of the lens if they are to be in focus in the final image. This is illustrated in the laser diagram above. Just as a lens has a front focal point it also has a back focal point that is positioned behind the lens at the same distance as the front focal point and it is this back focal point which enables us to view the wavelengths from our object in the form of a 'diffraction pattern'

So what exactly are we seeing in the diffraction pattern? The first important principle to understand is the following:-

Note: When two waves of equal wavelength are superimposed in phase they give rise to constructive interference, in which the resultant amplitude is equal to the sum of the component amplitude. When added in anti phase they give destructive interference.



Diffraction Pattern



When two wavelengths of light are identical they will superimpose constructively upon each other increasing the overall height of both waves, as another identical wave comes along the over-all height will increase proportionally and so on. The greater the number of identical waves of light the larger the wavelength will become. It is this that allows us to separate regular repetitive backgrounds from fingerprint impressions.

The background pattern has a regular repetitive frequency, one wave would be represented as a small indistinct spot in the diffraction pattern, the next wave will superimpose on the first brightening it in the diffraction pattern, the next will do likewise and so on. The larger the background pattern the brighter the spot (or frequency spike) will become. The fingerprint impressions however has an irregular pattern which will not be so distinct in the diffraction pattern. It is now a simple task to isolate the background pattern frequencies and remove them while leaving the fingerprint intact. In fact it is also worth noting that although we may have removed the frequency spikes we have only reduced the effect of the background pattern we have not entirely removed it.

To understand this further we need to look at the diagram of the laser on the previous page (this is something you can set up and try for yourself, but remember never look directly at the laser and where appropriate wear eye protection)

In the laser diagram I am using the convex lens to perform my Fourier transform. I have previously used slide film to photograph a fingerprint impression on a very heavily spotted background (I. E. Map book or screen printed cardboard). I have now mounted it into a slide and placed it in front of the laser, it is identified in the diagram as the 'object'. The laser is a 1 milliamp red laser over which I have placed a small diverging lens on the laser to allow the light to spread over the fingerprint impression.

The light waves now pass through the object and through the convex lens, the lens subsequently bends the rays of light to the focal point and from there it forms an image. To see the diffraction pattern I have placed a piece of glass in the back focal plane of the lens and behind it I have placed a piece of black cardboard. The diffraction pattern will now be visible on the piece of cardboard, to remove the frequency spikes all I need to do is to place a black spot on the glass where the frequency spikes are located. The black paper can now be removed and the image that will be formed will no longer contain the background pattern. The diffraction pattern that this image will produce can be seen in the diffraction pattern above. The other effect that you will notice is that everything in the diffraction pattern is symmetrical around the centre spot and the pattern will be backwards to the image that you see in the slide. If the original image has lines in the horizontal they will be represented in the vertical in the diffraction pattern and vice versa. Also if you have a large pattern such as the spots the number of spots in the diffraction pattern will be few and a small number of spots in your image will be represented by many spots in the diffraction pattern.

The frequency spikes

Imagine having just the laser and placing two pieces of black cardboard on either side of the beam of light so as to just let through a single slit of light. The above diagram gives you a closer look at what is happening with the laser and the far viewing screen. I have used a laser as this is a coherent monochromatic light source with a parallel beam containing only a very limited number of wavelengths (frequencies). The viewing screen now clearly shows three distinct spots which relate to a specific frequency in the original image. The wavelength of light has been split into three each of which now appear in the diffraction pattern as distinct intensity spots. However this basic set up has problems as strictly speaking the diffraction pattern is only located at infinity and it will also cause phase distortion. The answer to this problem is the convex lens as it will bring the plane waves into focus in it's focal plane.

The intensity of the spots will increase proportionally if more frequencies are present (another light source) the centre spot will always have a greater intensity than the surrounding spots.,

The distance of the spots from the central spot can now be pre-determined mathematically.

If the angle θ fulfils certain requirements then the following equation is true :-

$$s = f q \lambda$$

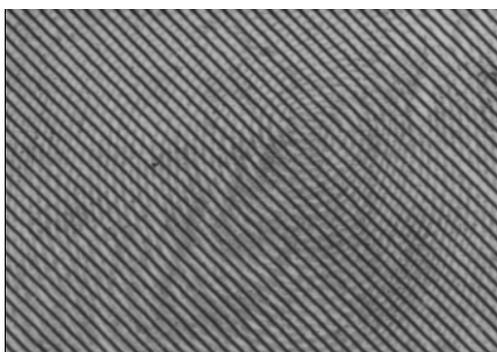
(s = separation of the spots, λ = wavelength of light, f = focal length of the lens and q = spatial frequency)

Today these scientific principles have been developed into the fast Fourier transform algorithm which allows the computer to use complex mathematical algorithms to calculate the frequency spectrum. This is generally displayed in a graphical form as a plot of magnitude against frequency.

It should therefore follow that if we have a fingerprint image with a regular background pattern we can use the Fourier transform to convert our image into a diffraction pattern. Once it is in this format we can clearly distinguish between the background pattern it' self and the rest of the image.

The background pattern will be visible as a series of frequency spikes which surround the centre spot (optical axis) and they will always be symmetrically positioned on either side of the centre spot. These spikes can subsequently be individually selected and removed with the use of a 'hanning' filter or a circular cut filter.

These changes can be applied to the original image so that the selected frequencies are removed without causing damage to the rest of the image. If too many frequencies are removed the image will eventually become unrecognisable which is why we only ever work on a duplicate of an original [authenticated image](#).



Ninhydrin on a bank note



Ninhydrin on a bank note after
FFT

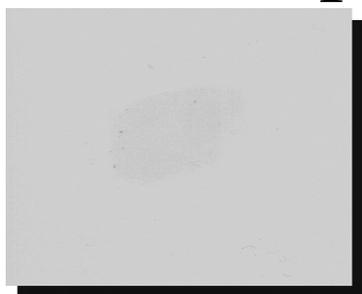
The images above clearly show the impact that the FFT technique has on fingerprint enhancement . In fact in my Police force we have significantly increased the number of identifiable fingerprints recovered from crime scenes with this technique.

However it is only the beginning of what digital technology can do for the fingerprint world, there are many other digital enhancement techniques which also significantly improve the quality of our fingerprint evidence.

It should also be noted that the results are only as good as the operator and it is important that law enforcement agencies using this technology ensure that their staff are given the appropriate training to ensure that this technique is used correctly within agreed protocols and procedures to achieve the desired results

The next page shows several different fingerprint impressions that have benefited from the application of the Fourier technique.

Spatial Filtering

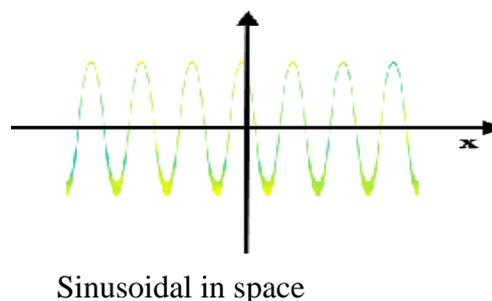
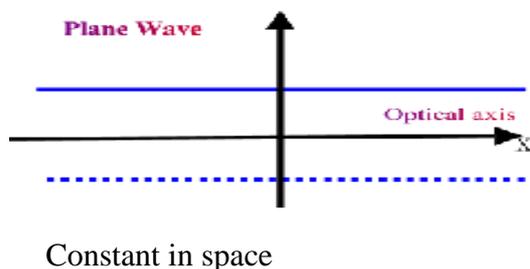


The Fourier transform technique can also be used to improve the contrast within an image. The two images above are photographs of a fingerprint in grease on a piece of glass. The fingerprint on the left is the mark as it originally appeared. The fingerprint on the right is the same fingerprint after enhancement with the Fourier transform. So how exactly do we achieve this?

If we were conducting research in the field of microscopy this would be called dark field illumination. It is achieved by changing the amplitude of the wavelength of light. To understand how this works we need to look again at our far field diffraction pattern.

The diffraction pattern shown here on the top left shows a bright spot at the centre of the image. This is representative of the optical axis and a section of every wavelength contained within the image runs through here. It is for this reason that any alteration performed on this central point must be done with the greatest of care. It is also why when you remove the background pattern by removing the frequency spikes, you never completely remove the wavelengths, rather you substantially reduce their effect on the overall image.

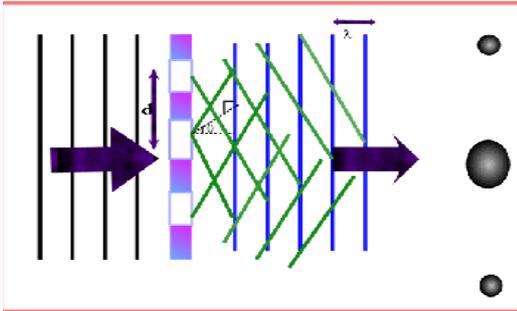
The output of light from an object can be broken down into a constant in space standing wave component and a sinusoidal-in-space standing wave component, both of which are equally spaced on either side of the optical axis, look at the following diagram :-



The standing wave corresponds to the spot at the centre of the diffraction pattern. It has been attenuated in amplitude relative to the incident wave. That is to say that the input wave changed at the output stage according to the amplitude transmittance of the object. Where T_a + Transmittance amplitude, E = electromagnetic energy (light) and A = amplitude the transmittance amplitude is calculated from the following equation

$$T_a = E_{out}/E_{in} = A_{out}/A_{in}$$

he two spots on either side are produced by the sinusoidal in space component. As they propagate away from the object they give rise to two plane waves, which travel symmetrically away from the object with respect to the optical axis. The angle is also predetermined by the number of cycles of a wavelength of light during a set time period, this is commonly referred to as the spatial period of the object. They subsequently produce two equispaced spots of light on either side of the centre spot. The far field diffraction pattern is therefore a combination of the constant in space and sinusoidal in space components. Look at the following diagram :-



I'm sure you can see how this diagram relates to the far field diffraction pattern. It is also why the pattern is symmetrically placed around the centre spot. Hence if you remove a spike from one side it is automatically removed from the other.

Therefore to alter the overall contrast of the image all we need to do is alter the amplitude of the centre spot. To do this in our DCS software we perform the following functions :-

1. Select the Fourier transform option and perform a forward Fourier transform
2. Enlarge the diffraction pattern to 200% and select a small circular area of interest in the centre of the centre spot.
3. Select the option box in the Fourier transform window and select the hanning filter set at 50% with the Nyquist frequency preserved.
4. Click OK and apply the change.
5. Inverse the Fourier transform.
6. Your image is now ready for normal contrast adjustment.

It should be noted that the smaller the percentage the more the filtering operation will remove, if the value is set too low it will over compensate. But if you do make a mistake it is a simple operation to undo the Fourier transform and start again.